Object Tracking in a 2D UWB Sensor Network

Cheng Chang
Electrical Engineering
and Computer Science
University of California, Berkeley
Email: cchang@eecs.berkeley.edu

Anant Sahai
Electrical Engineering
and Computer Science
University of California, Berkeley
Email: sahai@eecs.berkeley.edu

***Abstract*— We consider object tracking by a UWB sensor network using multi-path measurements in different scenarios: single Tx with single Rx, multiple Tx with single Rx, and multiple Tx with multiple Rx. For each scenario, we examine the Cramér-Rao lower bound (CRLB) for the high-SNR case where multi-path measurements are corrupted by iid Gaussians. We focus on the dense network asymptotics and show how the CRLB is inversely proportional to the number of measurements available to the network as a whole, even when the individual measurements are taken from different locations. An order-optimal semi-linear algorithm is given.**

I. Introduction

Rather than putting tags or localizers on the objects of interest, we envision using physical layer information from a wireless network to track the positions of objects in the environment.[5] While the positions of objects may be useful information for its own sake, the wireless communication channel is also largely determined by the objects in the sensor network field. Collectively tracking the position of the objects could in turn help the channel estimation problem, which has the potential to benefit the sensor network's communication system.

This problem is related to multi-static radar. Position estimation using TOA (time of arrival) or TDOA (time difference of Arrival) in a radar network has been studied in [1] and [2], and the accuracy is discussed in [3]. The Cramér-Rao lower bound (CRLB) of the position estimation is also studied in [1]. The CRLB for unbiased estimation of Td (TOA) is studied in [4]:

$$\delta^2_{Td} \geq \frac{N_0}{\int_0^T [\frac{\partial A(t)}{\partial t}]^2 dt} \tag{1}$$

Where $A(t)$ is the received signal, T is the observation interval, N_0 is the noise power spectral density, δ^2_{Td} is the estimation variance of Td. The important thing to observe is that the CRLB is inversely proportional to the received signal energy. This energy can increase by either using more powerful pulses or by using longer durations and taking averages over time.

Multistatic radar systems are usually expensive and sparsely deployed. As such, their asymptotic behavior is not extensively studied. In contrast, sensors or personal wireless devices may be both cheap and densely deployed in the field, and thus the asymptotic behavior of the CRLB is interesting. Throughout, we denote the CRLB of x as $V(x)$. The key question is whether "spatial averaging" (by taking independent measurements from different locations) improves accuracy in the same way that "time averaging" does (by taking independent measurements from the same location).

In this paper we focus on a two dimensional UWB network where transmitters and receivers have known locations on the plane. In a UWB sensor network, transmitters can send out signals with very high bandwidth. We assume a specular reflection model for the wireless environment in which signals are simply delayed, phase-shifted, and attenuated by the objects they encounter. If the A/D converters have very high sampling rates, it is possible to estimate the path-lengths of the reflections from the channel response. We focus on the high SNR regime in which the channel response is known well and so the multi-path distance measurements are assumed to be corrupted by small iid Gaussian noises.[1]

First we study the single transmitter single receiver case. We show that this is impractical since the normalized Cramér-Rao lower bound of the estimation problem is very large even with a motion model. Sensor pairs are not enough to track objects.

With more sensors, an object can be tracked. We study the dense network asymptotics of the CRLB while each transmitter/receiver pair maintains the same SNR.

[1] As such, we are also implicitly assuming that we can separate the channel response into a "foreground" relating to the objects of interest and a "background" that is coming from aspects of the environment that are not changing or uninteresting for other reasons.

With uniformly distributed sensor nodes, the asymptotic CRLB is inversely proportional to the total number of measurements available. With N transmitters and M receivers, this corresponds to $O(\frac{1}{NM})$ for centralized processing. For processing done only at a single receiver, the CRLB behaves like $O(\frac{1}{N})$. In addition to the lower bounds, we give an order-optimal semi-linear algorithm that operates by nonlinearly transforming the problem of position-estimation into a linear least-squares problem.

Finally, we explore the case of multiple objects. The challenge is to sort the multi-paths between different sensor pairs into sets corresponding to a single physical object. We propose a two step heuristic centralized algorithm. The first step is inspired by the Hough Transform[7] and it coarsely estimates object positions and associates multi-path measurements with objects. The semi-linear algorithm then refines the position estimates using the associated measurements.

II. Single Object Tracking

A. Single Transmitter Single Receiver Network

With only a single multi-path length d (the distance from Tx to the object plus the distance from the object to the Rx) available, it is impossible to estimate the position of the object. Suppose only Tx 3 was available in Fig.2. d simply specifies an ellipse $d_2(p, Tx_3) + d_2(p, Rx) = d$, where d_2 is the Euclidean distance. In bi-static radar, this ellipsoidal ambiguity is resolved by using Doppler or angle of arrival information.[1] Such information is not likely to be available in a sensor network scenario.

It is possible to estimate the position of the object up to the natural 4-fold symmetry if we have a very good motion model. We parameterize the motion as (x_0, y_0, x_N, y_N) and assume constant velocity between the two endpoints. In [6], we gave an algorithm which can consistently estimate the motion in principle. However, this problem is practically unsolvable since the CRLB is huge. Fig.1 illustrates this for motion toward the origin and shows $N\max(V(x_0), V(y_0), V(x_N), V(y_N))$ on a logarithmic scale. To achieve decent performance would require taking prohibitively many measurements N.

B. Large Sensor Networks

Fig.2 illustrates the situation in a large sensor network from the perspective of a single receiver. If we can simultaneously measure all the multi-path distances, then ellipse laceration can give us the object's position.

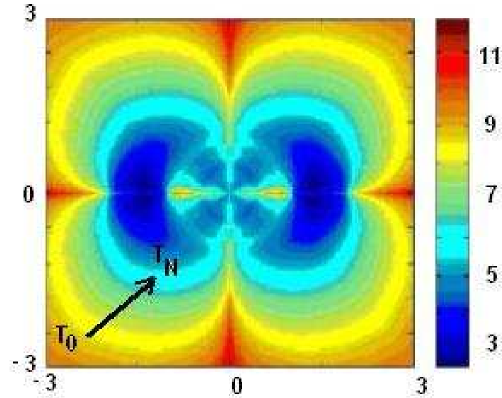

Fig. 1. Normalized CRLB bound $\log_{10}(N\max(V(x_0), V(y_0), V(y_N), V(y_N)))$ when $(x_0, y_0, x_N, y_N) = (x, y, x(1-\frac{1}{\sqrt{x^2+y^2}}), y(1-\frac{1}{\sqrt{x^2+y^2}}))$ Tx, Rx are at (-1,0) and (1,0) respectively

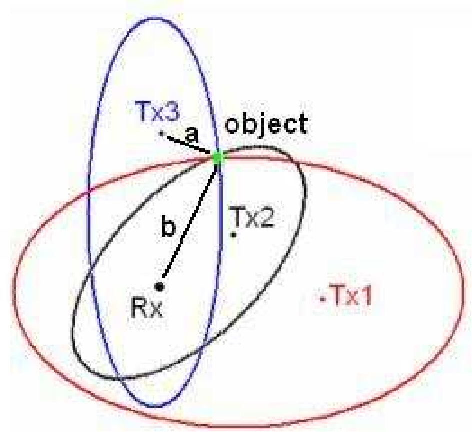

Fig. 2. Multiple sensors and a single object. Ellipse laceration gives position.

1) Cramér-Rao Bound: Consider a sensor network with N transmitters with known positions $(x_i, y_i), i = 1, ...N$ and M receivers with known positions $(x_j', y_j'), j = 1, ...M$. Let (x, y) denote the unknown position of the object. The observation vector $\overrightarrow{l} = (l_{11}, l_{12}, ...l_{ij}, ..., l_{NM})$, consists of the multi-path measurements. Assume the observations are corrupted by iid Gaussian noises $\sim N(0, \sigma^2)$, then $l_{ij} = \sqrt{(x-x_i)^2 + (y-y_i)^2} + \sqrt{(x-x_j')^2 + (y-y_j')^2} + \varepsilon_{ij}$. From this, we can derive the Fisher Information Matrix $J_{2\times 2}$ corresponding to estimating (x, y). The Cramér-Rao lower bounds are $V(x) = J_{11}^{-1}, V(y) = J_{22}^{-1}$. In our model, J is a random matrix where the randomness comes from the random placement of the sensors.

We are interested in the asymptotic behavior of the CRLB as the sensor network gets dense. In [6], we showed that the CRLB converges in probability. In this short paper, we simply state the results for six different scenarios. The first three (a) correspond to centralized processing with the sensor nodes uniformly distributed inside a circle of radius r. The last three (b) correspond to decentralized processing at a single receiver. These results capture the geometry of the problem.

a.1 Object at the origin:

$$\lim_{N\to\infty, M\to\infty} (V(x)+V(y))NM = 2\sigma^2 \text{ in probability}$$

We write the above equation in shorthand as $\boxed{V(x) + V(y) \approx \frac{2\sigma^2}{NM}}$.

a.2 Distant object away from the sensors at a distance of $L, L >> r$. This gives $\boxed{V(x) + V(y) \approx \frac{\sigma^2}{NM}\frac{2L^2}{r^2}}$. The CRLB goes to infinity as the object gets distant.

a.3 Same setup as (a.2) but in polar coordinates (ρ, θ), where $(x, y) = (\rho cos(\theta), \rho sin(\theta))$. This gives $\boxed{V(\rho) \approx \frac{\sigma^2}{4NM}, V(\theta) \approx \frac{2\sigma^2}{NMr^2}}$.

b.1 Object within sensor field with transmitters angularly uniformly distributed around (x, y): $\boxed{V(x) + V(y) \approx \frac{8\sigma^2}{3N}}$.

b.2 Distant object like in (a.2) except with processing at a single receiver yields: $\boxed{V(x) + V(y) \approx \frac{\sigma^2}{N}\frac{4L^2}{r^2}}$.

b.3 Same same setup as (b.2), except in polar coordinates: $\boxed{V(\rho) \approx \frac{\sigma^2}{4N}, V(\theta) \approx \frac{4\sigma^2}{Nr^2}}$.

The non-asymptotic behavior can be seen through simulations of the CRLB placing the receiver at the origin and the transmitters at regular spacings on the unit circle. We plot the normalized CRLB $N(V(x) + V(y)$ that corresponds to scaling down the transmit power so that the total received SNR is constant. As can be seen by comparing Fig.3 and Fig.4, the CRLB is indeed inversely proportional to the total number of the measurements.

2) A semi-linear algorithm: To complement the CRLB, we give a semi-linear object tracking algorithm illustrated in Fig.5. Consider a single transmitter-receiver pair: $T_x = (a_i, b_i), R_x = (u_j, v_j)$. Write $l_{ti} = \sqrt{(x - a_i)^2 + (y - b_i)^2}$, the distance from the object to the i-th transmitter and $l_{rj} = \sqrt{(x - u_j)^2 + (y - v_j)^2}$, the distance from the object to the j-th receiver. If the multi-path distance is d_{ij}, it satisfies:

$$d_{ij} = \sqrt{(x - a_i)^2 + (y - b_i)^2} + \sqrt{(x - u_j)^2 + (y - v_j)^2}$$

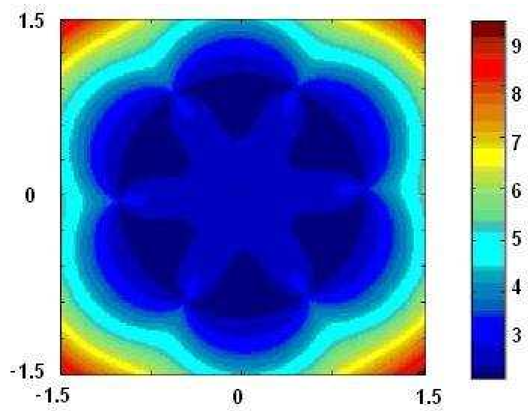

Fig. 3. Normalized Cramér-Rao bound, $N = 6$

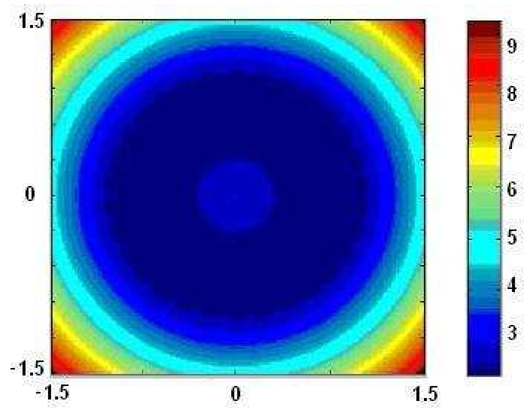

Fig. 4. Normalized Cramér-Rao bound, $N = 30$

which implies:

$$\begin{aligned} &(u_j - a_i)x + (v_j - b_i)y - d_{ij}\sqrt{(x - a_i)^2 + (y - b_i)^2} \\ &= \frac{u_j^2 + v_j^2 - a_i^2 - b_i^2 - d_{ij}^2}{2} \\ &(u_j - a_i)x + (v_j - b_i)y - d_{ij}l_{ti} = \\ &\frac{u_j^2 + v_j^2 - a_i^2 - b_i^2 - d_{ij}^2}{2} \end{aligned} \quad (2)$$

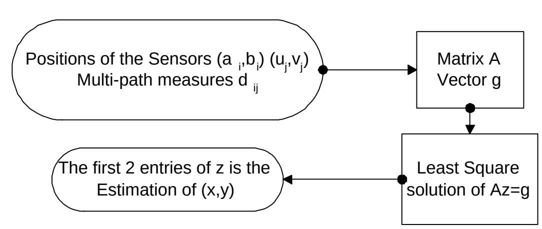

Fig. 5. Flowchart of the semi-linear algorithm

Similarly, we have :

$$(a_i - u_j)x + (b_i - v_j)y - d_{ij}l_{rj} = \frac{a_i^2 + b_i^2 - u_j^2 - v_j^2 - d_{ij}^2}{2} \quad (3)$$

We have $2NM$ linear equations of $N+M+2$ unknowns $\vec{z} = (x, y, l_{t1}, l_{t2}, ...l_{tN}, l_{r1}, ...l_{rM})^T$. Assume $N + M \geq 4$ and write them as:

$$A\vec{z} = \vec{g} \quad (4)$$

where the $2((i-1)M + j) - 1$ entry of the $2NM$ dimensional vector $\vec{g}$ is $\frac{u_j^2+v_j^2-a_i^2-b_i^2-d_{ij}^2}{2}$, and the $2((i-1)M + j)$ entry is $\frac{a_i^2+b_i^2-u_j^2-v_j^2-d_{ij}^2}{2}$. The A matrix is similarly defined by (2) and (3). The least-squares $\vec{z}$ is:

$$\vec{z} = (A^T A)^{-1} A^T \vec{g} \quad (5)$$

In [6], we proved that the estimation variance of the above algorithm is order-optimal when the sensors are all both transmitters and receivers. The estimation variances for x and y in that case are $\approx \frac{28\sigma^2}{3N^2}$, where N is the number of sensors. In Fig.6, we show some simulation results of the semi-linear algorithm in the decentralized case when it is used by a single receiver with 30 transmitters. In Fig.7, we plot the squared error and the CRLB together — this illustrates how the semi-linear algorithm achieves the same $O(1/N)$.

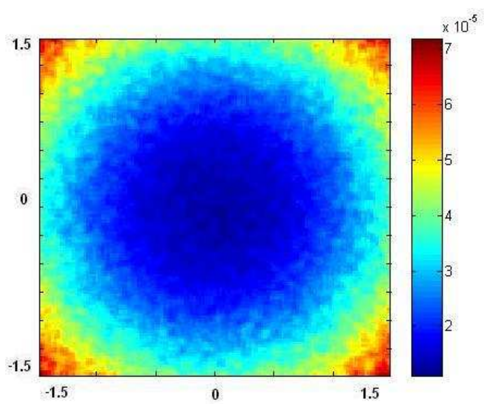

Fig. 6. Squared Error $(\hat{x} - x)^2 + (\hat{y} - y)^2$ for $N = 30$,$\sigma^2 = 10^{-4}$. 50 Monte-Carlo trials per point were used to simulate.

III. Multiple Objects in a Sensor Network

In a sensor network with N transmitters, M receivers and L objects, each transmitter-receiver pair has L paths that it sees. In order to estimate the positions of the objects, we need to first associate each multi-path distance measure with an object. There are $(L!)^{NM-1}$ different

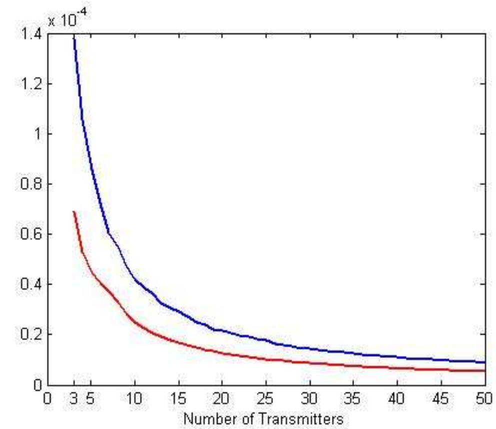

Fig. 7. Blue curve on top: Squared Error $(\hat{x} - x)^2 + (\hat{y} - y)^2$ Decentralized semi-linear algorithm. 5000 trials per point.
Red curve on bottom: CRLB $V(x) + V(y)$ for $(x, y) = (0.5, 0.5)$,$\sigma^2 = 10^{-4}$

associations making exhaustive search impractical. We propose the following heuristic centralized algorithm which is inspired by the Hough Transform[7].

First, we discretize the region of interest and then use the measured distances to assign scores to grid points. The score function for point (x, y) is defined as:

Definition 1 (Score function $S(x, y)$ of (x,y)):

$$S(x, y) = \sum_{i,j} \max(1 - \frac{S_{i,j}(x, y)}{K}, 0) \quad (6)$$

where K is a tunable parameter and $S_{i,j}(x, y) = \min_k(\sqrt{(x - a_i)^2 + (y - b_i)^2} + \sqrt{(x - a_j)^2 + (y - b_j)^2} - d_k(i, j))^2$ for all transmitter-receiver pairs (i, j) and all multipaths k measured between i and j. The minimizing k for each (i, j) is associated to the point (x, y).

After assigning a score to each grid point, we search for points with high scores. Finally we can use the linear algorithm for single object to estimate the positions of the objects using the associated distance measurements. A simulation is illustrated in Fig.8 and Fig.9.

IV. Conclusions and Future Work

In this paper, we studied object tracking in a sensor network. We argued that a single transmitter, single receiver network can not accurately estimate the position of an object since the Cramér-Rao lower bound is very big even with a severe constraint on the motion. If the object moves in a smooth way, then it is impossible to track its motion in a single transmitter, single receiver network because two different smooth motions can yield the same multi-path measurements.[6]

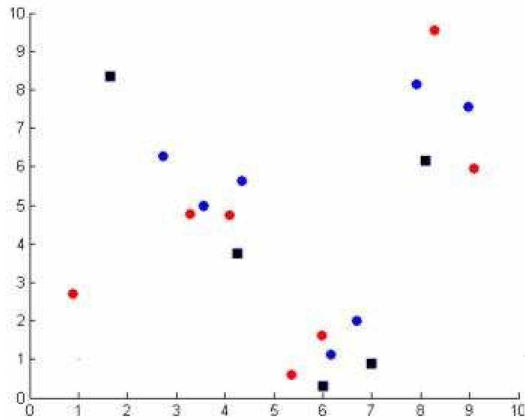

Fig. 8. 5 objects are located at black boxes, red and blue dots are transmitter and receivers respectively

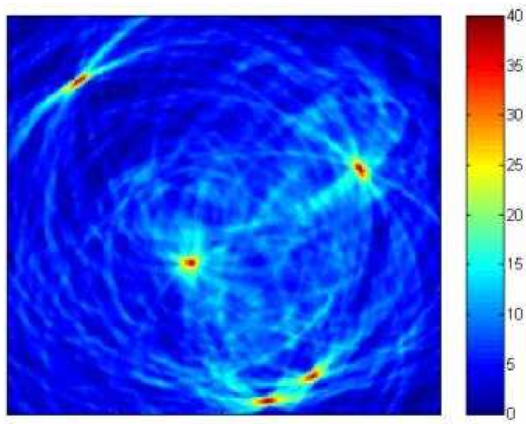

Fig. 9. Scores $S(x, y)$ on (x, y)

For a dense sensor network, we computed the CRLB in different scenarios and analyzed the asymptotic behavior of the CRLB as the number of sensors goes to infinity. It turns out that the estimation accuracy is proportional to the total number of measurements available. The interesting result here is that the quality of the bound depends on the coordinates that we use. For objects within the sensor field, it makes sense to use Euclidean coordinates. For distant objects, polar coordinates give a better sense of the asymptotic performance. In addition to the lower bounds, we gave a semi-linear algorithm which is order optimal and can work in both the decentralized and centralized estimation scenarios. Finally, we examined the case of multiple objects in the sensor field and presented a heuristic algorithm for that case that works well in simulations.

In our paper, we always assume that the object sizes are small and so we considered them as points on the plane. In practice, objects are not negligibly small, especially in the context of a dense sensor network. How to take the sizes of the objects into account needs to be studied. In an indoor environment, the structural objects (walls, floors, ceilings, etc.) also present a challenge. These could be on the same scale as the entire sensor network. In the same vein, the specular reflection based model here is a bit dubious. Experimental work needs to be done to see how well it fits. Even so, we believe that the basic lesson of the paper is sound — that dense UWB-based networks can combine multipath information from different pairs of transmitters and receivers to do estimation of the environment.

We also restricted attention to the high SNR regime where the multi-path distance measurements are accurate. In the low SNR regime, the path lengths cannot be extracted from a single channel estimate. We conjecture that the estimation accuracy will depend primarily on the total signal energy available. It may be possible to work directly with score functions in the style of Fig.9 and calculate scores based directly on the multipath responses rather than the extracted distances.

In a dense wireless network, the pairwise channels are not independent. With accurate object position information, we might estimate motion and thereby jointly predict the communication channels much better than is possible using only a single channel response. This brings up interesting theoretical questions regarding the nature of coherence time in UWB networks and raises the possibility of a deeper relationship between capacity and tracking accuracy.

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